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Possible Origin of Extra States in Particle Physics

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The transformation of a discrete state into resonances is considered in the framework of The Friedrichs model. The number of resonances depends on the form factor which describes the interaction of the discrete state and the continuum and for a reasonable form this number exceeds the usual one-to-one correspondence. The physical implications of the phenomenon are discussed.

Hadron spectroscopy is one of the most intricate areas of modern particle physics. Theoreticians and experimentalists are competing with each other in interpreting already discovered resonances and inventing new types. The pragmatic approach to the problem of classification of resonances consists in identifying a standard, nonexotic set of particles which admits a conventional quark model interpretation and then considering the superfluous states as candidates for exotic ones—glueballs, hybrids, molecules etc. Indeed, the direct observation of exotic quantum numbers forbidden in the usual quark model simplifies the discovery these new kinds of matter, but until now very few examples of open exotics exist, and even those are not confirmed. Also, it is generally believed that new kinds of mesons should have specific decay modes and specific creation processes, e.g., glueballs should easily be created in decays of J/ψ and have large couplings with $\eta\eta$, $\eta\eta'$, and $\eta'\eta'$ channels and suppressed electromagnetic ones (Gershtein *et al.*, 1984). Unfortunately, these statements have only qualitative character because in the case of hidden exotics, mixing with quark states may drastically change branching ratios into different channels (Amsler and Close, 1995, 1996a, b).

So, we return to the usual procedure of separating superfluous states with nonexotic quantum numbers as the main tool for discovery of new kinds of mesons (a similar problem for baryons also needs discussion). In this

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situation we must have the firm statement that the number of states which we predict in the framework of some model (e.g., potential, bag, string, etc.) is not influenced by interaction. In the present paper we analyze the validity of the hypothesis that there is always a one-to-one correspondence between the number of predicted states and the number of resonances. It is generally believed that the interaction with the decay channel provides the width of the resonance; it may shift the mass, but never changes the number of states. We show that this common point of view (we also were believers until recently) is generally wrong. In a simple but rather universal model we demonstrate that as a rule the number of states which is defined by interaction is greater than that expected, and only the weak-coupling regime gives a one-to-one correspondence.

We begin our discussion with the motivation of the model which will be used in this paper for the description of the unstable particles. Switching on the interaction of one particle with two (or few) others, whose total mass permits the transition on the mass shell, makes this particle unstable. This statement is trivial for any particle physicist. On the other hand, from the point of view of field theory, the question is not as simple as it seems. As an example, let us consider the theory of two scalar fields $\phi(x)$ and $\psi(x)$ with masses M and m, respectively, with

$$M > 2m \tag{1}$$

Now, if we switch on among others the interaction described by the vertex

$$S_{\text{int}} = \lambda \int d^4x\, \phi(x)\psi^2(x) \tag{2}$$

the theory becomes unstable. The latter means that the usual asymptotic conditions fail in this case. Of course, we can still calculate the Green functions in this theory, and investigate the complex singularity which corresponds to the initially stable ϕ-particle, but this is only part of the story. The most important questions from the physical point of view are as follows:

- What are the asymptotic states in this field theory?
- How can we calculate the S-matrix for these asymptotic states?

A general discussion of these questions in the relativistic case is rather complicated and we do not dwell on them here, referring the reader to Antoniou *et al.* (n.d.-a). Here we will give a schematic consideration, sufficient for qualitative description of many physical situations, where the nonrelativistic approximation is valid.

In terms of field theory, the direct consequence of the instability condition (1) is the nonvanishing interaction due to (2) for $t \to \pm\infty$, which could be established, e.g., in the interaction picture. The nonvanishing part of the interaction requires the redefinition of the asymptotic Hamiltonian (which usually is taken equal to the free one) and the true asymptotic states should be defined as eigenstates of this modified Hamiltonian. The correct scattering theory should be considered now for these true asymptotic states due to the residue (well behaved at $t \to \pm\infty$) interaction. At first sight it seems that the whole problem becomes technically very difficult (new Feynmann diagrams with new nonlocal propagators, vertexes, etc.), but this is not really true. To see this the path integral approach is very useful. In this approach it is obvious that the basic object is the total action and that the perturbation theory with respect to the free Hamiltonian makes the scattering theory ill-defined. So, if we use as asymptotic states the true ones, we can use the usual Feynmann diagrams for internal parts of the processes; the modification concerns only external lines.

Now, after this very short and schematic general introduction we shall start considering the subject of the present paper—a possible picture of asymptotic states, their properties, and the correspondence with the common point of view on the resonances in particle physics. The appropriate framework for the description of resonances is provided by the Friedrichs (1948) model, the touchstone of the general theory of perturbation of unbounded operators, which goes back to the late 1940s. This model is rather popular even nowadays, but unfortunately not among particle physicists. To simplify formulas we will not consider the second-quantized version of the Friedrichs model, which is directly connected with the asymptotic Hamiltonian of the field theory with interaction in the form of (2), limiting ourselves to its lowest level.

Let us denote by $|1\rangle$ the discrete state with energy ω_1 and by $|\omega\rangle$ the state with continuous spectrum for 0 to ∞. These states span the space of states of our system $\mathcal{H}$. The scalar products of the basis states are

$$\langle 1|1\rangle = 1 \qquad (3)$$
$$\langle \omega|\omega'\rangle = \delta(\omega - \omega')$$

The unperturbed Hamiltonian in $\mathcal{H}$ can be written as follows:

$$H_0 = \omega_1|1\rangle\langle 1| + \int_0^\infty d\omega\, \omega|\omega\rangle\langle\omega| \qquad (4)$$

Now let us add to H_0 a perturbation which describes the transitions between $|1\rangle$ and $|\omega\rangle$:

$$H_{\text{int}} = \lambda \int_0^\infty d\omega\, [f(\omega)|\omega\rangle\langle 1| + f^*(\omega)|1\rangle\langle\omega|] \qquad (5)$$

where λ is a coupling constant and $f(\omega)$ is a smooth, square-integrable function, which satisfies the condition

$$\omega_1 > \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega} \tag{6}$$

The physical sense of (6) will be clear later. As is seen from (4) and (5), our model describes exactly the situation discussed in the Introduction: the discrete state $|1\rangle$ is the state predicted in some model (potential, bag, string, etc.). The Hamiltonian

$$H = H_0 + H_{\text{int}} \tag{7}$$

describes the interaction of this state with the continuum.

Now $\omega_1 > 0$, to make the decay possible on the mass shell. The positioning of the threshold at the origin is not essential; we placed it there for simplicity.

The eigenvalue problem for the Hamiltonian (7) is

$$(H - E)\Psi(E) = 0 \tag{8}$$

and we have to solve it in the space $\mathcal{H}$, i.e., we shall seek the eigenvector $\Psi(E)$ in the following form:

$$\Psi(E) = \psi(E)|1\rangle + \int_0^\infty d\omega\, \psi(E, \omega)|\omega\rangle \tag{9}$$

where $\psi(E)$ and $\psi(E, \omega)$ are the unknown amplitudes for which, making use of (3) and (8), we obtain the system of equations

$$(\omega_1 - E)\psi(E) + \lambda \int_0^\infty d\omega\, \psi(E, \omega) f^*(\omega) = 0 \tag{10}$$

$$(\omega - E)\psi(E, \omega) + \lambda\psi(E) f(\omega) = 0$$

To solve this system, let us begin with the second equation and express $\psi(E, \omega)$ via $\psi(E)$:

$$\psi(E, \omega) = A\delta(\omega - E) - \frac{\lambda f(\omega)}{\omega - E}\psi(E) \tag{11}$$

where A is an arbitrary constant. Note that the first term in the r.h.s. of (11) arises because the factor $\omega - E$ in the equation has as a function of E a real zero at $E = \omega$. This expression for $\psi(E, \omega)$ can be used in the first equation (10), and we finally obtain the equation for the amplitude $\psi(E)$:

$$\left[\omega_1 - \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega - E}\right]\psi(E) = -\lambda A f^*(E) \tag{12}$$

Note that in this form equation (12) is only symbolic. Initially we considered system (10) for real values of E. The factor in square brackets on the left-hand side of (12) could be defined as the boundary value of the analytic function

$$\eta^{-1}(E) = \omega_1 - E - \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega - E} \tag{13}$$

This function, as is seen from its representation, has a cut $[0, \infty)$ and for real energy we can define its value from above and from below the cut:

$$\eta_\pm^{-1}(E) = \omega_1 - E - \lambda^2 \int_0^\infty d\omega \frac{|f|^2(\omega)}{\omega - (E \pm i\epsilon)} \tag{14}$$

These two functions $\eta_\pm(E)$ correspond to two different solutions of our eigenvalue problem (8)—in-going and out-going waves. So the proper form of equation (12) for real energy is

$$\eta_\pm^{-1}(E)\psi_\pm = -\lambda A f^*(E) \tag{15}$$

We see that the $\psi(E)$ [as well as $\psi(E, \omega)$] also acquires the subscript $\pm$.

In the mathematical literature the function $\eta(E)$ on the whole complex plane E is called the partial (or one-particle) resolvent of H. For the particle physicist the more familiar term is the Green function or the propagator. The solution of (15) can be written in the form

$$\psi_\pm = \psi^0 - A\eta_\pm(E)\lambda f^*(E) \tag{16}$$

where $\psi^0(E)$ is the solution of (15) with vanishing r.h.s. The latter depends upon the properties of the resolvent $\eta(E)$: if it has a pole on the first sheet on the real axis, then

$$\psi^0 = B\delta(E - E_0) \tag{17}$$

where E_0 is the position of the pole. Close inspection of equation (14) shows that this pole may exist only below a threshold. From the physical point of view it seems rather pathological and to prevent creation of this unwanted pole it is sufficient to impose condition (6) on the form factor $f(\omega)$. If this is done, then the first term in (16) is absent, and gathering together (8), (11), and (16), we obtain the final form of eigenvector $\Psi(E)$:

$$\Psi_\pm(E) = \left\{ |E\rangle + \lambda f^*(E)\eta_\pm(E)\left[|1\rangle + \lambda \int_0^\infty d\omega \frac{f(\omega)}{\omega - (E \pm i\epsilon)}|\omega\rangle \right]\right\} \tag{18}$$

This formula is the key point of our present discussion and therefore we must carefully investigate it and its consequences.

First, the most important fact that follows from (18) is that the Hamiltonian of our system (7) has only continuous spectrum—the discrete state $|1\rangle$ has been dissolved in the continuum.[2] The comparison of eigenvectors of H_0 and H leads us to the conclusion that in the unstable case there is no analyticity in the coupling constant λ. To understand the fate of the discrete level with $E = \omega_1$ we must investigate the resolvent $\eta(E)$ on the complex plane E.

The common point of view on this question is the following—the pole at the point $E = \omega_1$ moves to the second sheet, acquiring negative imaginary part, and transforms into the Breit–Wigner resonance. This point of view is supported by calculations in the limit $\lambda \to 0$. Indeed, the inverse resolvent $\eta_+^{-1}(E)$ could be represented in the form

$$\eta_+^{-1}(E) = \omega_1 - E - \lambda^2(r(E) + i\pi|f|^2(E)) \tag{19}$$

where $r(E)$ is the real part of the integral on the r.h.s. of (13). If we assume that $r(E)$ and $|f|^2(E)$ are smooth functions in the vicinity of ω_1, then from (19) it follows that a new pole of the resolvent will be at the point

$$\begin{aligned} E_c &= \omega_1 - \lambda^2 r(\omega_1) - i\pi\lambda^2|f|^2(\omega_1) \\ &= \tilde{\omega}_1 - i\Gamma \end{aligned} \tag{20}$$

Note that the representation (19) is valid if we start from the upper rim of the cut and continue to the second sheet from above. We also can start from the lower rim of the cut and continue to the second sheet from below. There of course we will find the complex conjugated partner of (20).

This consideration is valid only for infinitesimal values of the coupling constant and cannot be applied even qualitatively for the case of hadronic resonances, where typical coupling with decay products is large. In this case we have to consider the equation for complex poles of the resolvent without approximation and the whole form factor $f(\omega)$ becomes important. To illustrate this, let us consider several examples that show that the result of switching off the interaction may lead to qualitatively unexpected consequences.

Example 1. Let us take the form factor $f(\omega)$ in the form

$$|f|^2(\omega) = \frac{\omega^{1/2}}{\omega + \rho^2} \tag{21}$$

where ρ is real. The inverse resolvent according to (13) is given by

$$\eta^{-1}(E) = \omega_1 - E - \lambda^2 \int_0^\infty d\omega \, \frac{\omega^{1/2}}{\omega + \rho^2} \frac{1}{\omega - E} \tag{22}$$

[2]This phenomenon has to be compared to the case when ω_1 lies below threshold. In this case equation (15) has the nonpathological homogeneous solution (17), and finally we obtain two eigenvectors of H—the perturbed discrete state and the perturbed continuous state.

and after integration we arrive at

$$\eta^{-1}(E) = \omega_1 - z^2 - \frac{i\pi\lambda^2}{z - i\rho} \tag{23}$$

where we have defined the variable $z = \sqrt{E}$ in such a way that the first sheet of the E plane corresponds to the upper half-plane of z and the second sheet of E to the lower half-plane of z. Condition (6) means in this case that

$$\omega_1 > \frac{\pi\lambda^2}{\rho} \tag{24}$$

which in turn implies that the equation $\eta^{-1}(E) = 0$ has the roots

$$z_{1,2} = \pm[\omega_1 - 2\gamma d - \gamma^2]^{1/2} - i\gamma$$
$$z_3 = -id \tag{25}$$

where d and γ are given by

$$\rho = d + 2\gamma, \qquad \pi\lambda^2 = 2\gamma(\omega_1 + d^2) \tag{26}$$

For the new parameters γ and d, inequality (24) reads

$$\omega_1 > 2\gamma d \tag{27}$$

Recall that (27) prevents penetration of z_i to the upper half-plane of z (or to the first sheet of E). As is seen from equations (25) and (27), here we can have two different situations:

- Two complex-conjugated poles on the second sheet of the E plane, with one antibound state, below the threshold, also on the second sheet: $\omega_1 - 2\gamma d > \gamma^2$.
- Three antibound states and no resonances: $\gamma^2 > \omega_1 - 2\gamma d > 0$.

The whole resolvent is

$$\eta(E) = \frac{z + i(d + 2\gamma)}{(z + id)[(z + i\gamma)^2 - (\omega_1 - 2\gamma d - \gamma^2)]} \tag{28}$$

and for $\lambda \to 0$ ($\gamma \to 0$) it has only two complex-conjugated poles

$$E_c = \omega_1 - 2\gamma d \pm 2i\gamma\omega_1^{1/2} + O(\lambda^4) \tag{29}$$

and no antibound states.

Example 2. In this case the form factor is given by

$$|f|^2(\omega) = \frac{\lambda^2\omega^{1/2}}{(\omega - \rho^2)(\omega - \rho^{*2})} \tag{30}$$

Now, ρ is a complex number. Proceeding as in the previous example, we obtain the inverse resolvent:

$$\eta^{-1}(E) = \omega_1 - z^2 + \frac{i\pi\lambda^2}{\rho - \rho^*} \frac{1}{(z + \rho)(z - \rho^*)} \tag{31}$$

where z is the square root of energy, defined as above. The condition on the parameters now is

$$\omega_1 - \frac{i\pi\lambda^2}{(\rho - \rho^*)|\rho|^2} > 0 \tag{32}$$

In this example the equation $\eta^{-1}(E) = 0$ has four solutions which correspond to the following situations:

- Two pairs of complex conjugated poles (resonances).
- Pair of complex conjugated double poles. [Here a fine tuning of parameters is needed (Antoniou *et al.*, n.d.-b): Re $\rho = \sqrt{\omega_1}$, Im $\rho = (\pi\lambda^2/16\omega_1)^{1/3}$.]
- One pair of complex poles and two antibound states.
- Four antibound states.

All these cases in the limit $\lambda \to 0$ fuse together at E_c

$$E_c = \omega_1 + \frac{\pi\lambda^2(\omega_1 - |\rho|^2)}{2\rho_2[(\omega_1 - |\rho|^2)^2 + 4\rho_2\omega_1]} - \frac{i\pi\lambda^2\sqrt{\omega_1}}{[(\omega_1 - |\rho|^2)^2 + 4\rho_2\omega_1]} + O(\lambda^4) \tag{33}$$

where $\rho_2 = \text{Im}\ \rho$.

We could continue this set of examples, but the general idea in now clear: as far as the consideration of singularities is fulfilled nonperturbatively, the number of resonances created out of stable states exceeds the expected one-to-one correspondence.

Questions which immediately arise in discussions with particle physicists are: How can the number of states be changed? Doesn't this contradict the completeness relation. To answer these questions and forestall more elaborate ones we first should prove the completeness of the solutions obtained and then treat the problem of discrete states in the case of resonances.

First let us fix the arbitrary constant $A = 1$ in the final expression for the eigenvector $\Psi(E)$, equation (18). Making use of normalization conditions (4) and the Sokhotsti–Plemel relation, one can prove that

$$(\Psi_{\pm}(E))^{+}\Psi_{\pm}(E') = \delta(E - E') \tag{34}$$

Further, the Sokhotski–Plemel relation provides us with the following equation for real E:

$$\eta_+(E) - \eta_-(E) = 2\pi i\lambda^2|f|^2(E)\eta_+(E)\eta_-(E) \tag{35}$$

Using (35), one can convince oneself that the following remarkable relation is valid:

$$\int_0^\infty dE\, \Psi_+(E)(\Psi_+(E))^+ = |1\rangle\langle 1| + \int_0^\infty d\omega\, |\omega\rangle\langle\omega| \tag{36}$$

The same is true also for the out-going solution $\Psi_-(E)$. Equations (34) and (36) tell us that the set of solutions $\Psi_+(E)$ [or $\Psi_-(E)$] forms a complete system in our case of unstable particle.[3] The other question, which is rather difficult to formulate precisely, concerns the status of resonances as a "particle" or "discrete state." This question has been intensively discussed in a series of papers of the Brussels–Austin group (Prigogine, 1992; Antoniou and Prigogine, 1993; Bohm and Gadella, 1989) and in a textbook by Bohm (1993). Unfortunately, a comprehensive discussion of this subject will lead to functional analysis, very far from particle physics, and therefore we again will just schematically present the general ideas.

Let us return to solution (18) of the eigenvalue problem [for definiteness we shall speak of the $\Psi_+(E)$ solution] and consider it as a function of complex energy. As we already know, the resolvent $\eta_{+(E)}$ which enters into the r.h.s. of (18) has a pole (poles) on the second sheet in the point (points), say E_c. The residue in this pole is proportional to the expression in the square brackets, taken at $E = E_c$. Let us denote it by $\Psi_+^G(E_c)$:

$$\Psi_+^G(E_c) = |1\rangle + \lambda \int_0^\infty d\omega\, \frac{f(\omega)}{\omega - E}\, |\omega\rangle\Big|_{E\to E_c} \tag{37}$$

where the continuation to the point E_c should be performed from above the real axis. The superscript G stands for Gamow. It is this state, being properly continued to the second sheet, that is the eigenvector of H with complex eigenvalue E_c, and there exists a generalized spectral decomposition of H where $\Psi_+^G(E_c)$ enter as a discrete state. It goes without saying that the last sentence is a heresy from the point of view of the Hilbert space formulation of quantum theory, but we have not worked in the Hilbert space from the very beginning, when we considered the Hamiltonians with continuous spectrum. Usually we do not pay too much attention to the difference between the Hilbert space and the rigged Hilbert space (the space where the operators

[3] It is useful to compare this state with the situation where the discrete level lies below the continuum.

with continuous spectrum is defined). The reason for this is probably the Dirac invention of bra and ket vectors, which enter into the formulas in a very symmetric way as far as the real spectrum is concerned. The general situation is nevertheless the following: if we consider operators with continuous spectrum, we may use as states the wave packets, which are good, square-integrable elements of some Hilbert space $\mathcal{H}$. But among these vectors we cannot find the eigenvectors of our operators and we must extend our space to also include nonnormalizable vectors if we want to construct the spectral decomposition of operators. This extended space Φ^+ is really the space of the functionals, not the functions (recall the popular example of the δ-function). The space of functionals Φ^+ should be supplied with the space of test functions Φ, where these functionals may be defined. In such a way there arises a rigged Hilbert space or Gelfand triplet of spaces (though it is more appropriate to say a trinity of spaces)

$$\Phi \subset \mathcal{H} \subset \Phi^+ \tag{38}$$

Now we can return to the Gamow vector and explain its place in the present construction. First we emphasize that the continuation to the complex point E_c in (37) should be performed starting from above the real axis (if we simply put $E = E_c$ in the integrand, the answer will be wrong). The obstacle for direct continuation is the contour of integration: to move E below the real axis we have to deform the path of integration to the complex plane, which is impossible because the state $|\omega\rangle$ is defined for real ω only. At this point let us recall that the state $\Psi_+(E)$ belongs to the space Φ^+. If we can find the appropriate space of test functions Φ such that the $\langle\phi|\omega\rangle$ (where $\langle\phi|$ belongs to Φ) could be analytically continued to the lower half-plane, we would be able to make the analytic continuation of (37). The space Φ which we need for this purpose does exist and its elements have the following form:

$$\Phi \supset \langle\phi| = \int_0^\infty d\omega\ \rangle\omega|\phi(\omega) \tag{39}$$

where the function $\phi(\omega)$ belongs to the space of Hardy class functions from above, i.e., functions which could be analytically continued to the lower half-plane.

The above discussion shows that in spite of the absence of resonances in the unity decomposition (36) we can construct the corresponding states in the rigged Hilbert space. Moreover, there exists a generalized spectral Hamiltonian which could be analytically continued in the rigged space in such a way that the resonances will explicitly enter it. We will not present this construction here and will devote the remainder of this paper to the discussion of physical consequences of our approach.

As is seen from the general expression for eigenstates and our examples, the function $f(\omega)$ plays a very important role in the formation of resonances, their masses, widths, and number. Certainly, this important object should be derived in the framework of fundamental theory—QCD—but in the present situation it is hardly possible and we have to introduce it phenomenologically. Therefore we must investigate if there are some general requirements on these functions which follow from quantum theory. One we have already used in our approach forbids the appearance of the stable state below the threshold by condition (6). Particle physicists have recognized in the form factors used in our examples the factor $\sqrt{\omega}$—the two-particle phase volume, which defines the vanishing of the transition amplitude of one scalar into a pair of scalars. In the general case the power of relative momentum $\sqrt{\omega}$ will be $l + 1$, where l is the relative orbital momentum. In addition, we imposed the requirement of square integrability of the form factor. Actually we can relax this condition—all our arguments hold even for form factors which vanish at infinity. As we see, these conditions leave too much room for different parametrizations of the form factor and there may arise the impression that among these different possibilities there also exists a case when the number of states coincides with the initial one. Unfortunately, this very case does not fit into the aforementioned conditions. Indeed, let us take the following form factor:

$$|f|^2(\omega) = \sqrt{\omega} \tag{40}$$

which does not vanish at infinity. To define the inverse resolvent, we need to make one subtraction in the dispersion integral (13) at some point $E = -E_0$, where $E_0 = \rho^2 > 0$. This subtraction of an infinite constant from the integral term may be absorbed into the infinite renormalization of ω_i in (13). After this renormalization we arrive at the following expression for $\eta^{-1}(E)$:

$$\eta^{-1}(E) = \omega_1^r - z^2 - i\pi\lambda^2(z - i\rho) \tag{41}$$

where we have used the notations from our first example and the superscript r indicates renormalized. The equation $\eta^{-1}(E) = 0$ now is quadratic and has exactly one pair of complex conjugate solutions. So, in principle we may have the desired one-to-one correspondence, but the price is the infinite renormalization in the model which is considered as a phenomenological one. In addition, this subtraction of the integral may be absorbed into the renormalization of a physical quantity only in the case of S-wave decay; for higher waves there is nothing to renormalize. Therefore we consider this possibility unsatisfactory.

The model which we have considered may be generalized for many channels and several discrete states to describe more realistic situation in particle physics—mixing of states via interaction with a mutual continuum.

The most important features of this generalization are the following: all discrete states dissolve in the continuum, the number of eigenstates of the Hamiltonian is equal to the number of different continuums, and the equation which defines the positions of the poles is common for all states—it becomes the equation for the poles of the determinant of the partial resolvent, but the intensities of different poles depend on the specific channel. If again we consider the meromorphic class of functions, the number of resonances exceeds one-to-one correspondence.

The model we have considered is rather general and universal, and QCD, as the fundamental theory of the strong interaction, should provide us with some prescription for the key object of our approach to the form factor $f(\omega)$. As we have already mentioned, the relativistic generalization of the Friedrichs model is also possible (Antoniou *et al.*, n.d.-a) and the role of the square of the form factor in this case is played by the spectral density of the propagator of the bound discrete state; therefore, in the realistic situation the use of exponential functions is hardly possible. Rather, one should use a function with the usual threshold singularity and meromorphic character of the appropriate complex plane and therefore this also should lead to qualitatively the same picture. On the other hand, one can argue that among the lowest multiplets we do not observe any doubling of states; all of them are very nicely described by single Breit–Wigner poles. That is true, but at the same time, when we consider the excited states, the situation changes rather drastically. Sometimes the hypothesis about extra states fits better than the single state. The example of the most advanced analysis of the resonance picture in the singlet channel 0^{++}, in the framework of K-matrix formalism with channels $K\bar{K}$, $\eta\eta$, $\eta\eta'$, and $\pi\pi$ (Anisovich *et al.*, 1994, 1996), shows that the number of states exceeds the quark model predictions. The most favorable interpretation of these extra states, of course, is the glueball one, but the appearance of pole splitting cannot be rejected. Certainly a clearer situation is in the isospin 1 states, because here we have no admixture of glueballs and here we find the region of masses 1450–1700 Gev with quantum numbers 1^{--} where different fittings give several states (Review of Particle Properties, 1994). Also, if the shape of the resonance differs from the usual Breit–Wigner one, it may be a reflection of several poles which are not well separated and more accurate measurement of the phase may clarify the interpretation.

The last point we want to mention in conclusion is the dependence of the shape of resonances on the channel even for the well-established ones. This phenomenon is very well known, but usually it is interpreted as experimental errors or the influence of different interactions of different decay products. A multichannel generalization of our approach, which we have not considered here, clearly shows that this dependence is another manifestation of form factors $f_i(\omega)$ which are different for different channels and can be used for its investigation.

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